Using technology to promote understanding in mathematics

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29 de junio de 2016

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Applying technology in the essential aspect of mathematics education: the comprehension of ideas.

Or how the authoring tool
DESCARTES
is used to promote understanding

Technology contributes to the improvement of education in many useful ways. For example, by making large amounts of information available to the teacher and the learner, or by presenting interactive exercises with random data that help the student acquire proficiency in applying formulas or algorithms.

However, the essential aspect of mathematics education is not acquiring information, or knowing when and how to apply algorithms, nor even proving theorems by logical reasoning, but the *comprehension of mathematical ideas*.

Our purpose is to demonstrate how the authoring tool DESCARTES can promote better *understanding of mathematical ideas* by teachers and students.

Plan of the presentation

- First, we justify the claim that understanding math ideas is the essential aspect of mathematics education.
- Next we describe briefly the authoring tool DESCARTES.
- Then we show some learning experiences with DESCARTES.
- Finally we explain how DESCARTES promotes understanding.

Formalism, proposed by David Hilbert (1862-1943), was the dominant philosophy of mathematics during most of the XXth century, with its emphasis on axioms and proof.

It proved disastrous for education, in the form of what was called *New Math* in elementary school and the *Bourbaki approach* in higher education. Most of the problems in present day mathematics education can be traced to those two movements.

In order to liberate math education from such influences need a more sensible and useful philosophy of mathematics.

The word *mathematics* comes from the greek Mathemata, and means *that which can be understood, and taught, because it is rational*. In most european languages, the word for mathematics has the same greek root, but in dutch and flemish the word is Wieskunde, the *science of certainty*. The word proposed by Simon Stevin (1548-1620) was Wieskunst, the *art of certainty*.

Mathematicians that have influenced our philosophy of mathematics.

Simon Stevin, David Hilbert, Felix Klein[3], Kurt Gödel, Hans Freudenthal[4], Morris Klein[4], Reuben Hersh[2]

We know very well that no *certain knowledge* can be claimed about the material world. But mathematics deals with mental objects that have well defined properties and about which *certain knowledge* is not only possible, but mandatory (although with some limitations, according to Gödel's incompleteness theorem).

The *raison d'être* of mathematics is precisely the certaintinty that they provide, even though it applies only to abstractions.

Certainty is obtained through *understanding*, and it is provided only by reason and by no other type of authority, either human or divine.

In mathematics we build abstract models to represent some aspects of the real world[1]. When these models are successful, they help us understand a part of the world, make predictions about it's behavior and even gain control over it.

Maybe we can't explain why this happens, but it is an undeniable fact that it happens with considerable frequency.

Such important fact must be learned by every citizen.
It is an essential aspect of the basic culture of humanity.

[1] Mathematical models are themselves part of reality and can be studied mathematically. This generates what is known as *pure mathematics*.

Hans Freudenthal

We should never think about what kind of mathematics a child can learn, but about the kind that can contribute to the development of her/his human dignity.

Why do we teach mathematics?

- Mathematics constitutes an important part of our culture.
- Mathematics are extremely useful. Our way of life depends both on ancient and very recent mathematical ideas.
- Mathematics promotes the use of reason, instead of authority, trickery or brute force, for solving problems.

Every time we introduce a new set of mathematical ideas to our students, we should answer these three questions for them and for ourselves:

- Why and when were these ideas developed?
- How are they used in modern life?
- How can we better *understand* them?

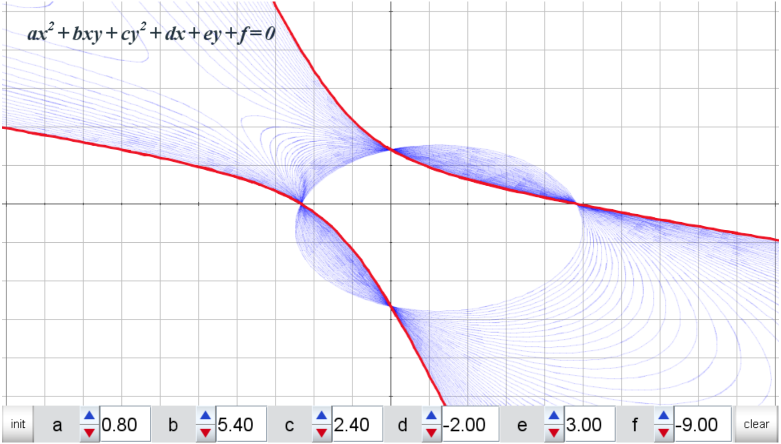
What is DESCARTES?

DESCARTES is an authoring tool, designed mainly for high school teachers of mathematics, to help them:

- *understand* the mathematical ideas they want to teach, and
- present these ideas in ways that help students *understand*.

DESCARTES is a programming environment with mathematical primitive objects: *parameters* (that can be manipulated by the student), *vectors, matrices, functions, algorithms* and *boolean conditions* (that can be programmed by the teacher) and *spaces*, both 2 and 3 dimensional, where *figures* and *graphs* are drawn and move when the parameters are manipulated by the student.

The graphs of 2nd degree equations in two variables are conic sections. Here the student controls 6 parameters: a, b, c, d, e, and f, and may study how the conic section behaves when the parameters are modified.



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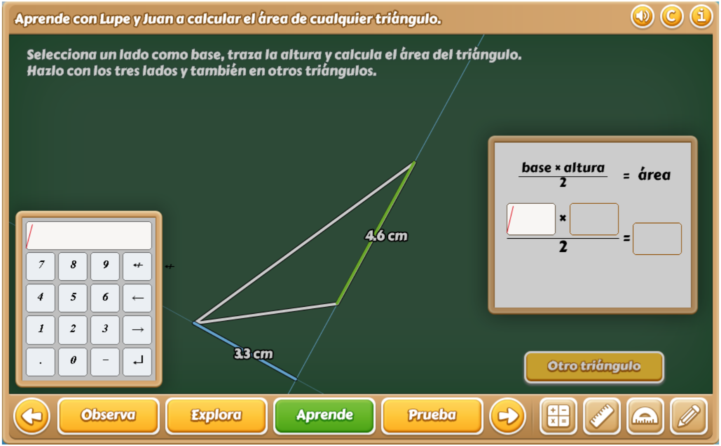
In what follows, we will show some examples of didactical units made with DESCARTES and, at the same time, describe some interesting experiences in *understanding* that occurred while they were being developed.

The example on *parabolic motion* is the main one we will use for this purpose, and the only to be analyzed in depth.

However we start with examples developed for the Elementary and for High Schools, in order to exhibit the breath of applications of the authoring tool DESCARTES.



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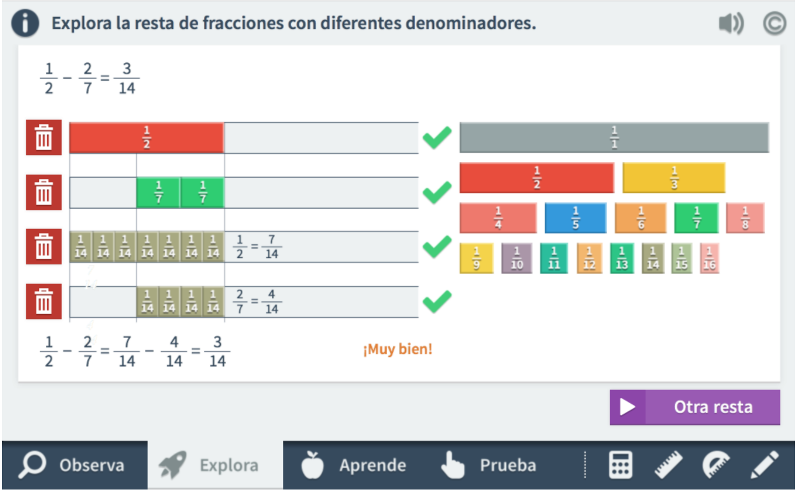
Interactive Learning Units for Elementary School Mathematics



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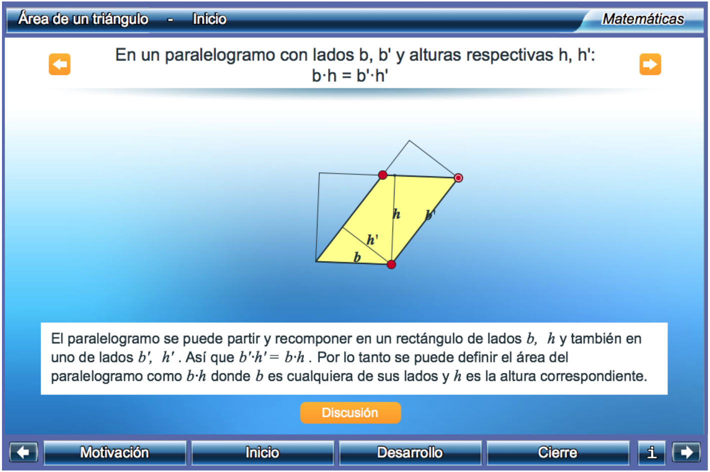
Basic Geometry: Areas of paralelograms and triangles



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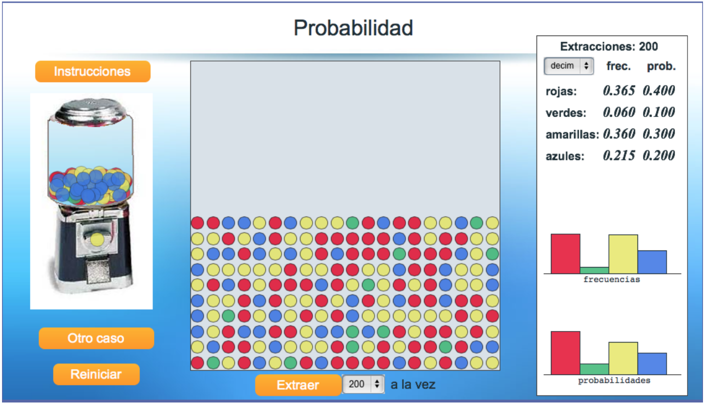
Random simulations: Probability



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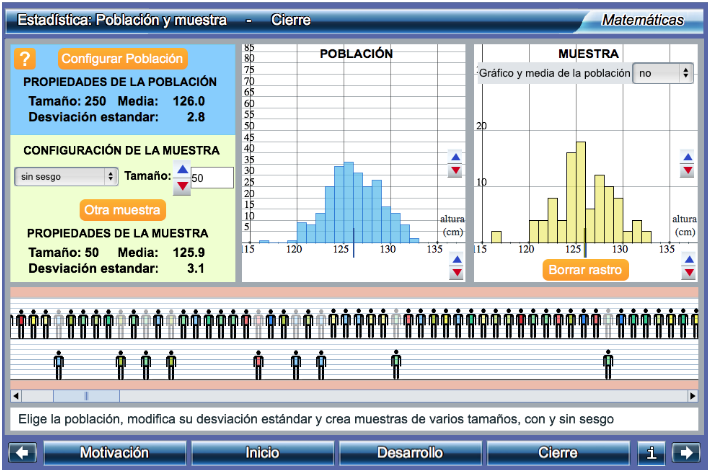
Random simulations: Population Sampling



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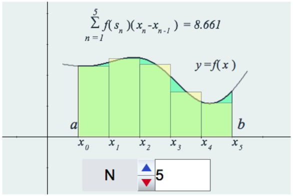
Cálculo Diferencial e Integral: Límites | **Introducción al Cálculo**

La integral, la derivada y el teorema fundamental del Cálculo

La integral

La integral de una función $f(x)$ en un intervalo $[a,b]$, se define de manera que corresponda al área bajo la gráfica de la función entre los puntos a y b del eje horizontal y se denota por:

$$\int_a^b f(x)dx.$$

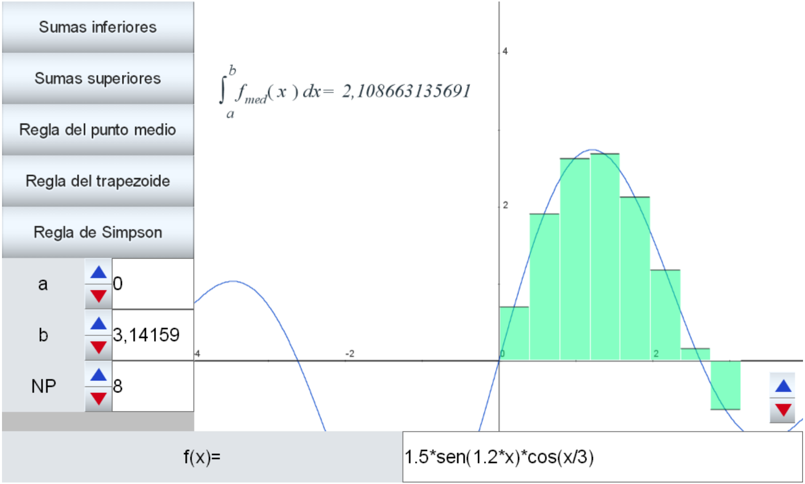


La definición formal se hace a través de un límite. Se considera una partición del intervalo $[a,b]$ que consiste de puntos $\{x_0, x_1, x_2, \ldots x_N\}$ tales que $a=x_0<x_1<x_2<\ldots<x_N=b$. En cada intervalo $[x_{n-1}, x_n]$ se escoge un punto s_n. La integral se define como el límite de las sumas de los productos de los valores $f(s_n)$ y las longitudes x_n-x_{n-1} de los intervalos $[x_{n-1}, x_n]$, cuando la partición se hace cada vez más fina, es decir, cuando el máximo de las longitudes x_n-x_{n-1} tiende a cero. En símbolos,

$$\int_a^b f(x)dx = \lim_{\|P\|\to 0} \sum_{n=1}^{N} f(s_n)(x_n-x_{n-1})$$

http://descartes.matem.unam.mx/recursos/Bachillerato/DGEE_DGTIC/

Numerical Integration



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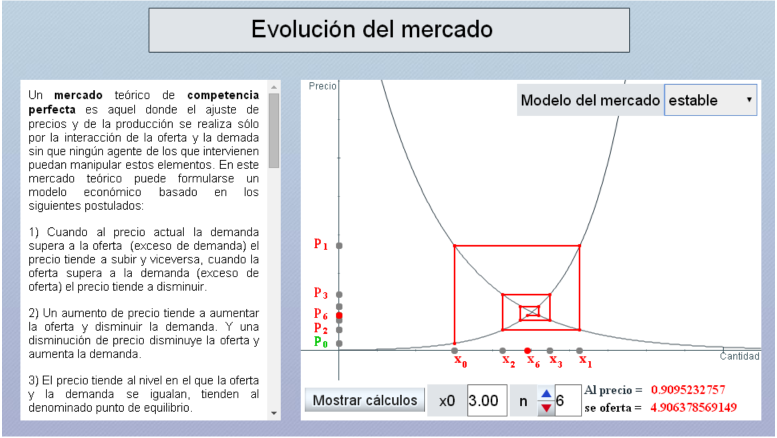

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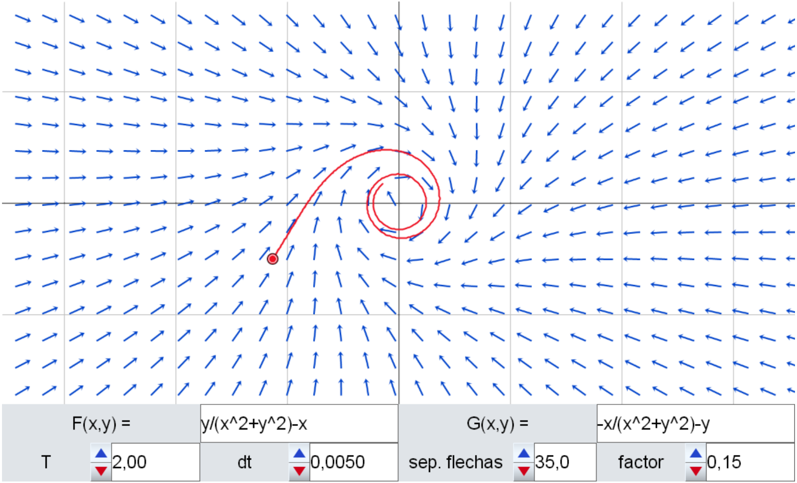
Autonomous Systems of ODE



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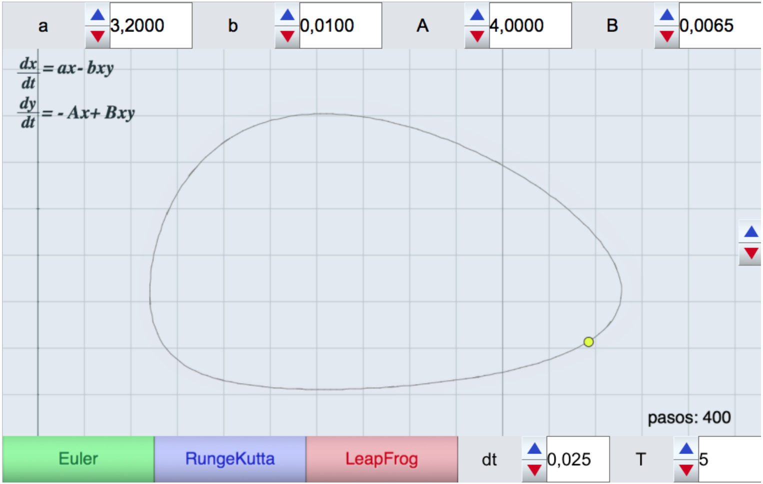
Lotka-Volterra



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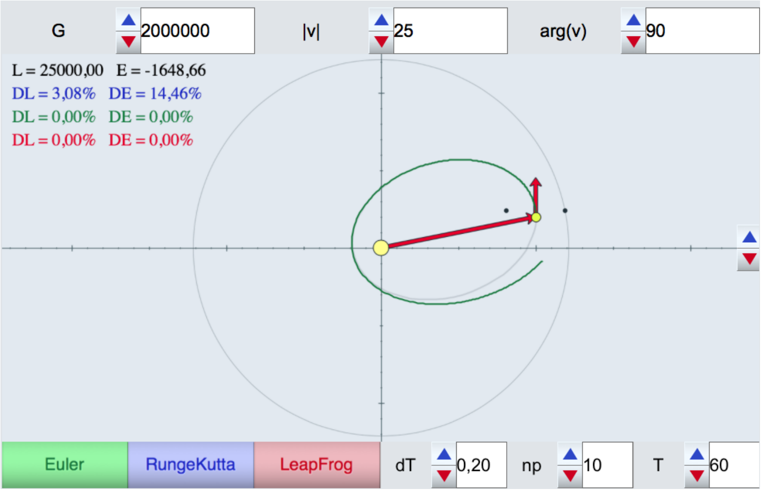
Kepler 2D (Leap-Frog)



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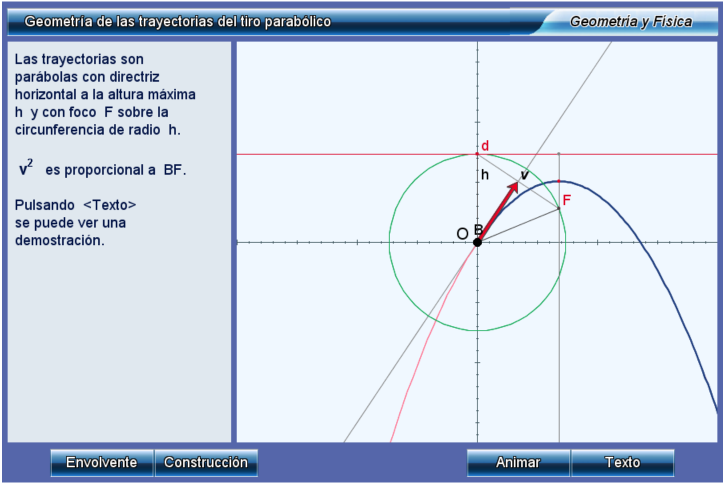

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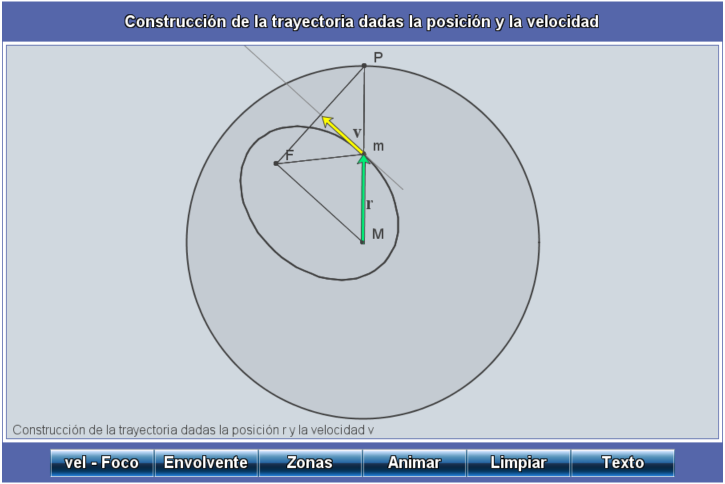
Kepler's Laws



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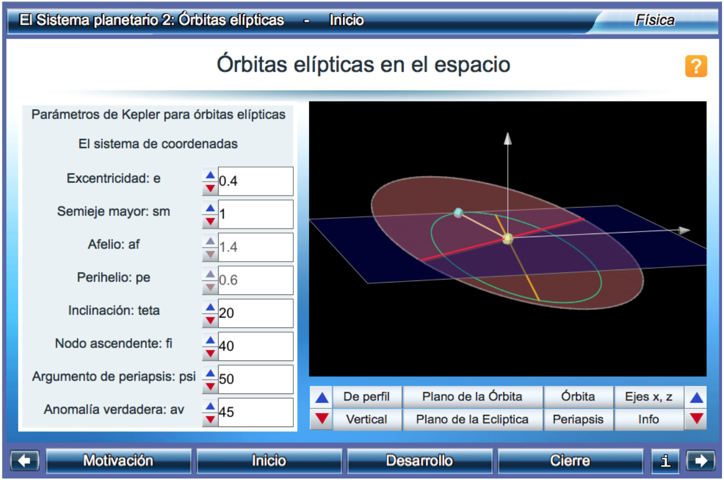

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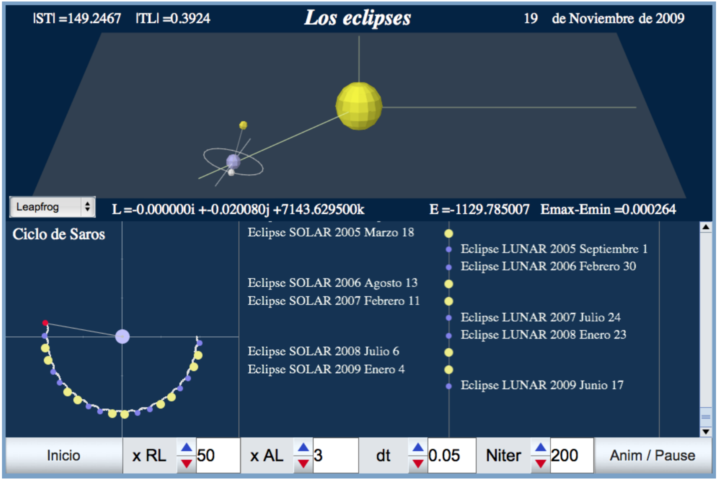

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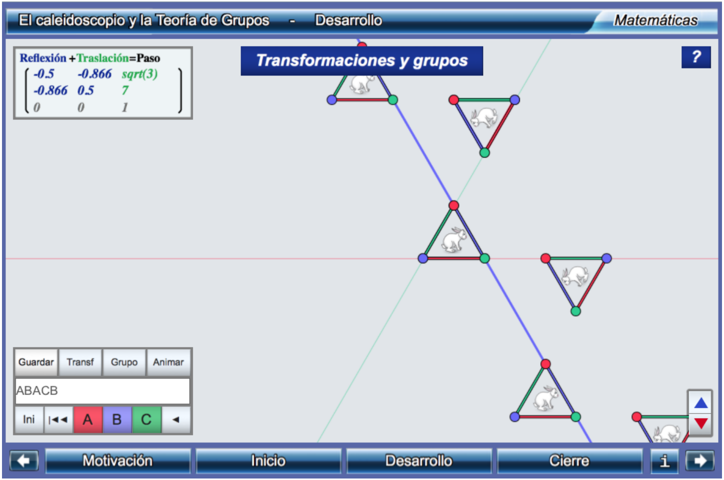

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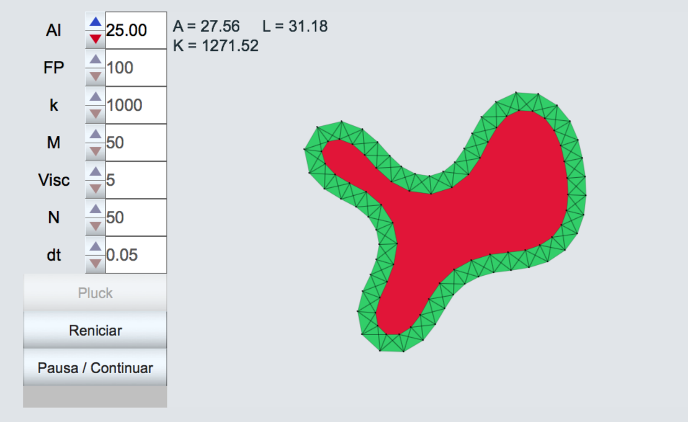
Research: The Red Cell 1



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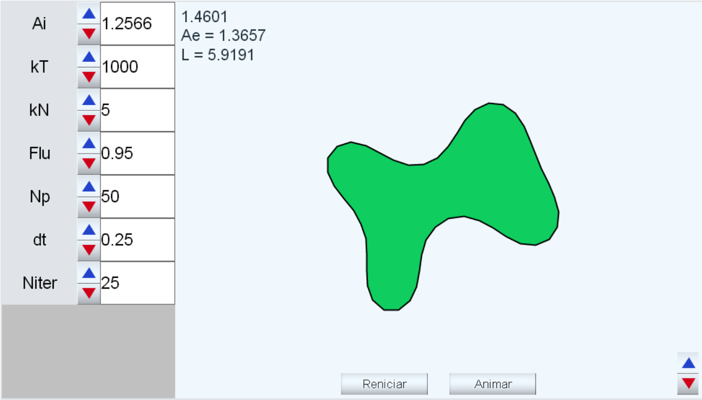
Research: The Red Cell 2



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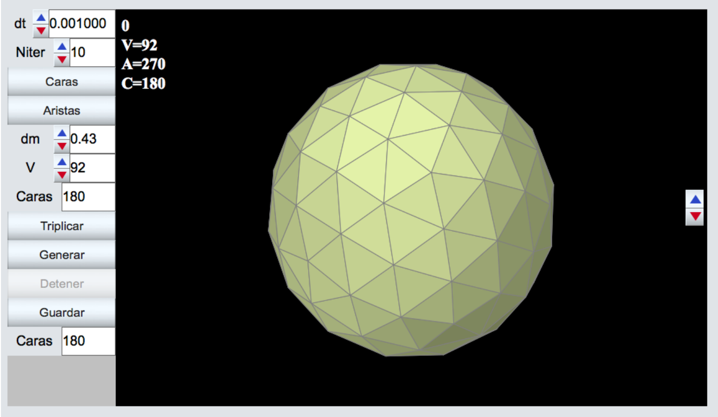
Research: Inscribed Polyhedra in the Sphere



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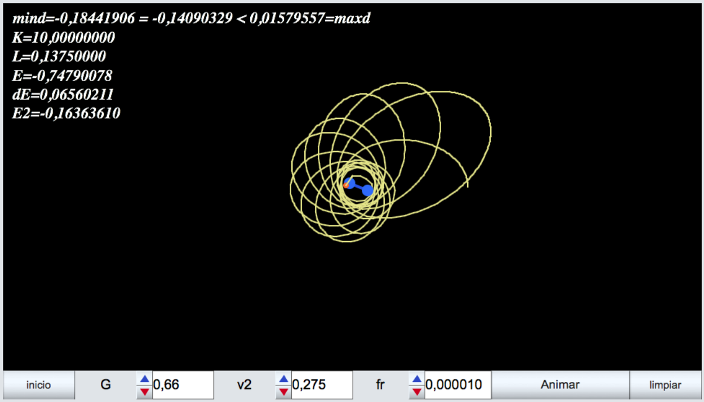
Research: Compound Celestial Bodies



http://arquimedes.matem.unam.mx/EJEMPLOS/04_EjemplosParaPosgrado/04_Cuerpos_celestes/

So... how come DESCARTES promotes understanding

- When a teacher develops a didactical unit with DESCARTES, he/she needs to plan and organize a teaching sequence. This helps her/him reflect on the subject matter's importance, usefulness and internal logic, things that should be reflected in the unit.
- Next, the teacher creates simulations that exhibit some mathematical ideas. In doing so, she/he may discover possible misconceptions and has the opportunity to reflect on them and make the necessary adjustments to deepen her/his understanding of the subject matter. With some luck, she/he may also discover new properties that were unknown to her/him or even to the rest of the world.

So... how come Descartes promotes understanding

- These activities make the teacher feel creative in his job, proud of her/his work and anxious to present it to her/his colleagues and students. Naturally she/he will be interested in discovering if the unit is useful, and in looking out for possible improvements. This improves the self esteem of the teacher, which is very beneficial for the educational process.
- During class, while the students are interacting with the unit in a computer or a tablet, the teacher is free to pay special attention to those who need individual motivation or explanations. This, in turn, provides valuable feedback to the teacher about the didactical units under study.

This is what happened in Spain's Proyecto EDA - Descartes
(look at the reported experiences in http://recursostic.educacion.es/eda/web/descartes/descartes_descartes2.html)
and continues to happen in Red Educativa Digital Descartes.

What can we recommend?

The development and use of interactive didactical units can be very profitable in mathematics education. However, these points must not be forgotten:

- It is necessary to involve the teachers in developing the units.
- Teachers must feel that these units are tools that they own and they can modify to suit their teaching needs.
- The units must be continuously renewed and improved.

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